

AXIALLY SYMMETRIC MULTISPHERALERONS IN YANG-MILLS-DILATON THEORY

Burkhard Kleihaus, and Jutta Kunz

Fachbereich Physik, Universität Oldenburg, Postfach 2503
D-26111 Oldenburg, Germany

February 1, 2008

Abstract

We construct sequences of axially symmetric multisphaleron solutions in $SU(2)$ Yang-Mills-dilaton theory. The sequences are labelled by a winding number $n > 1$. For $n = 1$ the known sequence of spherically symmetric sphaleron solutions is obtained. The solutions within each sequence are labelled by the number of nodes k of the gauge field functions. The limiting solutions of the sequences correspond to abelian magnetic monopoles with n units of charge and energy $E \propto n$.

Preprint hep-th/9609180

1 Introduction

Pure SU(2) Yang-Mills theory in 3 + 1 dimensions does not possess static, finite energy solutions. In contrast, SU(2) Yang-Mills-Higgs (YMH) theory possesses static, finite energy solutions, and so do SU(2) Einstein-Yang-Mills (EYM) and Yang-Mills-dilaton (YMD) theory.

YMH theory with a triplet Higgs field contains a stable static, spherically symmetric solution, the 't Hooft-Polyakov monopole [1], whereas YMH theory with a doublet Higgs field contains an unstable static, spherically symmetric solution [2, 3, 4]. This solution represents the electroweak sphaleron in the limit of vanishing Weinberg angle [4, 5]. For large Higgs boson masses the theory contains in addition a sequence of sphaleron solutions (without parity reflection symmetry) [6, 7].

EYM theory possesses a sequence of unstable static, spherically symmetric solutions for any finite value of the coupling constants [8, 9, 10]. Within the sequence the solutions are labelled by the number of nodes k of the gauge field function. When the dilaton field is coupled to the system, the solutions persist in the resulting Einstein-Yang-Mills-dilaton (EYMD) theory [11, 12, 13, 14, 15, 16]. Decoupling gravity leads to YMD theory with the corresponding sequence of unstable static, spherically symmetric solutions [17, 18].

In YMH theory, beside the 't Hooft-Polyakov monopole, there exist multimonopoles with magnetic charge $m = n/g$, where $n > 1$ is the topological charge or winding number and g is the gauge coupling constant. The spherically symmetric 't Hooft-Polyakov monopole has $n = 1$. Following a pioneering numerical study [19], axially symmetric multimonopoles have been obtained analytically in the Prasad-Sommerfield limit [20]. In this limit the energy of the multimonopoles satisfies the Bogomol'nyi bound $E = 4\pi n \langle \Phi \rangle / g$.

Similarly, beside the electroweak sphaleron, which carries Chern-Simons number $N_{CS} = 1/2$, there exist axially symmetric multisphalerons in YMH theory with Chern-Simons number $N_{CS} = n/2$ [21]. So it is natural to ask, whether analogous axially symmetric solutions exist also in EYM or YMD theory.

In this letter we construct multisphaleron solutions and their excitations in YMD theory. The appropriate axially symmetric ansatz for the multisphaleron solutions is analogous to the ansatz in YMH theory [19, 21, 22, 5]. Like the YMH multisphalerons, the YMD multisphalerons are labelled by an integer n , which represents a winding number with respect to the azimuthal angle ϕ . While ϕ covers the full trigonometric circle once, the fields wind n times around. For $n = 1$, spherical symmetry and the known sequence of YMD sphaleron solutions are recovered.

For each value of n we find a sequence of axially symmetric solutions, which can be labelled by the number of nodes k of the gauge field functions, analogous to the spherically symmetric case. For the limiting solutions, obtained for $k \rightarrow \infty$, we give an

analytic expression. The energies of the limiting solutions satisfy a Bogomol'nyi type relation $E \propto n$, which represents an upper bound for the energies of the solutions of the n -th sequence.

In section 2 we briefly review the lagrangian, discuss the ansatz and present the resulting energy functional. In section 3 we exhibit the multisphaleron solutions with $n \leq 4$ and $k \leq 4$. We discuss the limiting solutions in section 4 and present our conclusions in section 5.

2 Axially symmetric ansatz

Let us consider the lagrangian of YMD theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi \partial^\mu \Phi) - \frac{1}{2}e^{2\kappa\Phi} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) \quad (1)$$

with dilaton field Φ , SU(2) gauge field V_μ and field strength tensor

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu] , \quad (2)$$

and the coupling constants κ and g . This theory possesses a sequence of static spherically symmetric sphaleron solutions labelled by an integer k , which counts the number of nodes of the gauge field function [17, 18].

To obtain static axially symmetric multisphaleron solutions, we choose the ansatz for the SU(2) gauge fields analogous to the case of multimonopoles [19, 22] and electroweak multisphalerons [21]. We therefore define a set of orthonormal vectors

$$\begin{aligned} \vec{u}_1^{(n)}(\phi) &= (\cos n\phi, \sin n\phi, 0) , \\ \vec{u}_2^{(n)}(\phi) &= (0, 0, 1) , \\ \vec{u}_3^{(n)}(\phi) &= (\sin n\phi, -\cos n\phi, 0) . \end{aligned} \quad (3)$$

and expand the gauge fields ($V_\mu = V_\mu^a \tau^a / 2$) as

$$V_0^a(\vec{r}) = 0 , \quad V_i^a(\vec{r}) = u_j^{i(1)}(\phi) u_k^{a(n)}(\phi) w_j^k(\rho, z) , \quad (4)$$

whereas the dilaton field satisfies

$$\Phi(\vec{r}) = \Phi(\rho, z) . \quad (5)$$

Invariance under rotations about the z -axis and parity reflections leads to the conditions [19, 21, 5]

$$w_1^1(\rho, z) = w_2^1(\rho, z) = w_1^2(\rho, z) = w_2^2(\rho, z) = w_3^3(\rho, z) = 0 . \quad (6)$$

The axially symmetric energy functional

$$E = E_\Phi + E_V = \int (\varepsilon_\Phi + e^{2\kappa\Phi} \varepsilon_V) d\phi \rho d\rho dz \quad (7)$$

contains the energy densities

$$\varepsilon_\Phi = \frac{1}{2} [(\partial_\rho \Phi)^2 + (\partial_z \Phi)^2] \quad (8)$$

and

$$\begin{aligned} \varepsilon_V = & \frac{1}{2} \left[(\partial_\rho w_3^1 + \frac{1}{\rho}(nw_1^3 + w_3^1) - gw_1^3 w_3^2)^2 + (\partial_z w_3^1 + \frac{n}{\rho}w_2^3 - gw_2^3 w_3^2)^2 \right. \\ & \left. + (\partial_\rho w_3^2 + \frac{1}{\rho}w_3^2 + gw_1^3 w_3^1)^2 + (\partial_z w_3^2 + gw_2^3 w_3^1)^2 + (\partial_\rho w_2^3 - \partial_z w_1^3)^2 \right] . \quad (9) \end{aligned}$$

It is still invariant under gauge transformations generated by [19, 21]

$$U = e^{i\Gamma(\rho, z)\tau^i u_3^{i(n)}}, \quad (10)$$

where the 2-dimensional scalar doublet $(w_3^1, w_3^2 - n/g\rho)$ transforms with angle $2\Gamma(\rho, z)$, while the 2-dimensional gauge field (w_1^3, w_2^3) transforms inhomogeneously. We fix this gauge degree of freedom by choosing the gauge condition [21, 5]

$$\partial_\rho w_1^3 + \partial_z w_2^3 = 0 . \quad (11)$$

Changing to spherical coordinates and extracting the trivial θ -dependence (present also for spherically symmetric case $n = 1$) we specify the ansatz further [21]

$$\begin{aligned} w_1^3(r, \theta) &= \frac{1}{gr}(1 - F_1(r, \theta)) \cos \theta , & w_2^3(r, \theta) &= -\frac{1}{gr}(1 - F_2(r, \theta)) \sin \theta , \\ w_3^1(r, \theta) &= -\frac{n}{gr}(1 - F_3(r, \theta)) \cos \theta , & w_3^2(r, \theta) &= \frac{n}{gr}(1 - F_4(r, \theta)) \sin \theta . \end{aligned} \quad (12)$$

With $F_1(r, \theta) = F_2(r, \theta) = F_3(r, \theta) = F_4(r, \theta) = w(r)$, $\Phi(r, \theta) = \varphi(r)$ and $n = 1$ the spherically symmetric ansatz of ref. [18] is recovered.

The above ansatz and gauge choice yield a set of coupled partial differential equations for the functions $F_i(r, \theta)$ and $\Phi(r, \theta)$. To obtain regular solutions with finite energy density with the imposed symmetries, we take as boundary conditions for the functions $F_i(r, \theta)$ and $\Phi(r, \theta)$

$$\begin{aligned} r = 0 & : & F_i(r, \theta)|_{r=0} = 1, & i = 1, \dots, 4, & \partial_r \Phi(r, \theta)|_{r=0} = 0, \\ r \rightarrow \infty & : & F_i(r, \theta)|_{r=\infty} = F(\infty), & i = 1, \dots, 4, & \Phi(r, \theta)|_{r=\infty} = \Phi(\infty), \\ \theta = 0 & : & \partial_\theta F_i(r, \theta)|_{\theta=0} = 0, & i = 1, \dots, 4, & \partial_\theta \Phi(r, \theta)|_{\theta=0} = 0, \\ \theta = \pi/2 & : & \partial_\theta F_i(r, \theta)|_{\theta=\pi/2} = 0, & i = 1, \dots, 4, & \partial_\theta \Phi(r, \theta)|_{\theta=\pi/2} = 0 , \end{aligned} \quad (13)$$

(with the exception of $F_2(r, \theta)$ for $n = 2$, which has $\partial_\theta F_2(r, \theta)|_{\theta=0} \neq 0$ [23]) where $F(\infty) = \pm 1$ and $\Phi(\infty) = 0$. The boundary conditions for the gauge field functions at infinity imply, that the solutions are magnetically neutral. A finite value of the dilaton field at infinity can always be transformed to zero via $\Phi \rightarrow \Phi - \Phi(\infty)$, $r \rightarrow r e^{-\kappa \Phi(\infty)}$.

The variational solutions $F_i(\beta r, \theta)$ and $\Phi(\beta r, \theta)$, lead to $E_\beta = \beta^{-1} E_\Phi + \beta E_V$. Since the energy functional is minimized for $\beta = 1$, the virial relation [18]

$$E_\Phi = E_V \quad (14)$$

also holds for general n .

We now remove the dependence on the coupling constants κ and g from the differential equations by changing to the dimensionless coordinate $x = gr/\kappa$ and the dimensionless dilaton function $\varphi = \kappa\Phi$. The energy then scales with the factor $1/(\kappa g)$.

The dilaton field satisfies asymptotically the relation

$$\lim_{x \rightarrow \infty} x^2 \varphi' = D \ , \quad (15)$$

where D is the dilaton charge. The energy is related to the dilaton charge via

$$E = \frac{4\pi}{\kappa g} \lim_{x \rightarrow \infty} x^2 \varphi' = \frac{4\pi}{\kappa g} D \ . \quad (16)$$

3 Multisphaleron solutions

We solve the equations numerically, subject to the boundary conditions eqs. (13). To map spatial infinity to the finite value $\bar{x} = 1$, we employ the radial coordinate $\bar{x} = \frac{x}{1+x}$. The numerical calculations are based on the Newton-Raphson method. The equations are discretized on a non-equidistant grid in \bar{x} and an equidistant grid in θ , where typical grids used have sizes 50×20 and 100×20 covering the integration region $0 \leq \bar{x} \leq 1$ and $0 \leq \theta \leq \pi/2$. The numerical error for the functions is estimated to be on the order of 10^{-3} .

The energy density ε , defined by

$$E = \frac{1}{\kappa g} \int \varepsilon(\vec{x}) x^2 dx \sin \theta d\theta d\phi \ , \quad (17)$$

of the axially symmetric multisphaleron solutions and their excitations has a strong peak on the ρ -axis, while it is rather flat along the z -axis. Keeping n fixed and varying k , we observe, that the ratio of the maximum energy density ε_{\max} to the central energy density $\varepsilon(x=0)$ remains almost constant. Also, with n fixed increasing k , the location of the peak of the energy density approaches zero exponentially. On the other hand, with fixed k and increasing n the peak of the energy density moves outward. The

central energy density $\varepsilon(x=0)$ as well as the maximum energy density ε_{\max} are shown in Table 1 for the sequences $n = 1 - 4$ with node numbers $k = 1 - 4$. Also shown in Table 1 is the energy E .

In the following we exhibit as one example the multisphaleron solution for $n = 3$ and $k = 3$. In Fig. 1 we show the energy density ε . In Figs. 2a-d we show the gauge field functions F_i , which go from $F_i(0) = 1$ to $F_i(\infty) = -1$, passing three times zero in any direction. Finally, in Fig. 3 we show the dilaton function φ .

4 Limiting solutions for $k \rightarrow \infty$

The known sequences of solutions often tend to a simpler limiting solution. For the spherically symmetric YMD sequence with winding number $n = 1$, the limiting solution for $k \rightarrow \infty$ is given by [18]

$$w_\infty = 0, \quad \varphi_\infty = -\ln\left(1 + \frac{1}{x}\right), \quad (18)$$

and describes an abelian magnetic monopole with unit charge, $m = 1/g$. The gauge field functions w_k approach the limiting function $w_\infty = 0$ nonuniformly, because of the boundary conditions at the origin and at infinity, where $w_k = \pm 1$.

For the axially symmetric sequences with $n > 1$ we observe a similar convergence with node number k . As for $n = 1$, the numerical analysis shows, that the gauge field functions $(F_i)_k$ tend to the constant value zero in an exponentially increasing region. For $(F_i)_\infty = 0$, the set of field equations reduces to a single ordinary differential equation for φ_∞

$$\left(x^2 \varphi'_\infty\right)' - 2e^{2\varphi_\infty} \frac{n^2}{2x^2} = 0. \quad (19)$$

This yields the spherically symmetric limiting solution

$$(F_i)_\infty = 0, \quad \varphi_\infty = -\ln\left(1 + \frac{n}{x}\right), \quad (20)$$

corresponding to an abelian magnetic monopole with n units of charge. Thus the limiting solution of the sequence is charged, whereas all members of the sequence are magnetically neutral. This phenomenon is also observed in EYM and EYMD theory. (A detailed discussion of the convergence there is given in ref. [16].)

To demonstrate the convergence of the sequence of numerical solutions for φ_k to the analytic solution φ_∞ , we show the functions φ_k for $k = 1 - 4$ together with the limiting function φ_∞ in Fig. 4 for $n = 3$. The k -th function deviates from the limiting function only in an inner region, which decreases exponentially with k . The value $\varphi_k(x=0)$ decreases roughly linearly with k .

The energy of the limiting solution of the sequence n is given by

$$E = \frac{4\pi}{\kappa g} \int_0^\infty \left[\frac{1}{2} \varphi_\infty'^2 + e^{2\varphi_\infty} \frac{n^2}{2x^4} \right] x^2 dx = \frac{4\pi}{\kappa g} n, \quad (21)$$

i. e. the energy satisfies a Bogomol'nyi type relation, $E \propto n$. This relation is in agreement with eq. (16), since $D = n$ for the limiting solution. This limiting value for the energy, eq. (21), represents an upper bound for each sequence, as observed from Table 1. The larger n , the slower is the convergence to the limiting solution.

Further details on the solutions and the convergence properties of the sequences will be given elsewhere [23].

5 Conclusions

We have constructed sequences of axially symmetric multisphaleron solutions in YMD theory. The sequences are characterized by a winding number n , describing the winding of the fields in the azimuthal angle ϕ , while the solutions within each sequences are labelled by the node number k of the gauge field functions. For $n = 1$ the known spherically symmetric sequence is obtained.

The multisphalerons have a torus-like shape. The maximum of the energy density occurs on the ρ -axis. With fixed n and increasing k the maximum moves inward along the ρ -axis, whereas with fixed k and increasing n it moves outward.

For fixed n and $k \rightarrow \infty$ each sequence approaches an analytically given limiting solution. The limiting solution has vanishing gauge field functions and corresponds to an abelian monopole with n units of magnetic charge. Because of the coupling to the dilaton, the energy of this limiting solution is finite. It satisfies a Bogomol'nyi type relation with energy $E \propto n$.

The spherically symmetric YMD sphalerons possess fermion zero modes [24]. Since the electroweak sphaleron and multisphalerons have also fermion zero modes [25, 21, 26], we expect zero modes to be present also for the YMD multisphalerons.

The spherically symmetric sequence of $n = 1$ YMD sphaleron solutions can be continued in the presence of gravity, yielding a corresponding sequence of EYMD solutions, which still depends on a coupling constant [11, 12, 13, 14, 15, 16]. By continuity, we conclude, that the axially symmetric $n > 1$ YMD multisphaleron solutions also exist in the presence of gravity, representing axially symmetric EYMD solutions. In the limit of vanishing dilaton coupling constant, they should reduce to axially symmetric EYM solutions, and thus axially symmetric generalizations of the Bartnik-McKinnon solutions [8]. The outstanding question then is, whether there also exist the corresponding EYM and EYMD black hole solutions.

References

- [1] G. 't Hooft, Magnetic monopoles in unified gauge theories, Nucl. Phys. B79 (1974) 276;
A. M. Polyakov, Particle spectrum in quantum field theory, Zh. Eksp. Teor. Fiz. Pis'ma Red. 20 (1974) 430 [JETP Lett. 20, (1974) 194].
- [2] R. F. Dashen, B. Hasslacher and A. Neveu, Nonperturbative methods and extended-hadron models in field theory. III. Four-dimensional non-abelian models, Phys. Rev. D12 (1974) 4138.
- [3] J. Boguta, Can nuclear interactions be long-ranged? Phys. Rev. Lett. 50 (1983) 148.
- [4] F. R. Klinkhamer and N. S. Manton, A saddle-point solution in the Weinberg-Salam theory, Phys. Rev. D30 (1984) 2212.
- [5] B. Kleihaus, J. Kunz and Y. Brihaye, The electroweak sphaleron at physical mixing angle, Phys. Lett. 273B (1991) 100;
J. Kunz, B. Kleihaus, and Y. Brihaye, Sphalerons at finite mixing angle, Phys. Rev. D46 (1992) 3587.
- [6] J. Kunz and Y. Brihaye, New sphalerons in the Weinberg-Salam theory, Phys. Lett. B216 (1989) 353.
- [7] L. G. Yaffe, Static solutions of SU(2)-Higgs theory, Phys. Rev. D40 (1989) 3463.
- [8] R. Bartnik and J. McKinnon, Particlelike solutions of the Einstein-Yang-Mills equations, Phys. Rev. Lett. 61 (1988) 141.
- [9] N. Straumann and Z. H. Zhou, Instability of the Bartnik-McKinnon solutions of the Einstein-Yang-Mills equations, Phys. Lett. B237 (1990) 353.
- [10] D. V. Gal'tsov and M. S. Volkov, Sphalerons in Einstein-Yang-Mills theory, Phys. Lett. B273 (1991) 255.
- [11] E. E. Donets and D. V. Gal'tsov, Stringy sphalerons and non-abelian black holes, Phys. Lett. B302 (1993) 411.
- [12] G. Lavrelashvili and D. Maison, Regular and black hole solutions of Einstein-Yang-Mills dilaton theory, Nucl. Phys. B410 (1993) 407.
- [13] T. Torii and K. Maeda, Black holes with non-Abelian hair and their thermodynamical properties, Phys. Rev. D48 (1993) 1643.

- [14] P. Bizon, Saddle points of stringy action, *Acta Physica Polonica* B24 (1993) 1209.
- [15] C. M. O'Neill, Einstein-Yang-Mills theory with a massive dilaton and axion: String-inspired regular and black hole solutions, *Phys. Rev. D*50 (1994) 865.
- [16] B. Kleihaus, J. Kunz and A. Sood, SU(3) Einstein-Yang-Mills-dilaton sphalerons and black holes, *Phys. Lett. B*372 (1996) 204;
B. Kleihaus, J. Kunz and A. Sood, Sequences of Einstein-Yang-Mills-dilaton black holes, *Phys. Rev. D*54 (1996) .
- [17] G. Lavrelashvili and D. Maison, Static spherically symmetric solutions of a Yang-Mills field coupled to a dilaton, *Phys. Lett. B*295 (1992) 67.
- [18] P. Bizon, Saddle-point solutions in Yang-Mills-dilaton theory, *Phys. Rev. D*47 (1993) 1656.
- [19] C. Rebbi and P. Rossi, Multimonopole solutions in the Prasad-Sommerfield limit, *Phys. Rev. D*22 (1980) 2010.
- [20] P. Forgacs, Z. Horvath and L. Palla, Non-linear superposition of monopoles, *Nucl. Phys. B*192 (1981) 141;
M. K. Prasad and P. Rossi, Construction of exact multimonopole solutions, *Phys. Rev. D*24 (1981) 2182.
- [21] B. Kleihaus and J. Kunz, Multisphalerons in the weak interactions, *Phys. Lett. B*329 (1994) 61;
B. Kleihaus and J. Kunz, Multisphalerons in the Weinberg-Salam theory, *Phys. Rev. D*50 (1994) 5343.
- [22] N. S. Manton, Complex structure of monopoles, *Nucl. Phys. B*135 (1978) 319.
- [23] B. Kleihaus and J. Kunz, in preparation.
- [24] G. Lavrelashvili, Fermions in the background of dilatonic sphalerons, *Mod. Phys. Lett. A*9(94) 3731.
- [25] J. Boguta, and J. Kunz, Hadroids and sphalerons, *Phys. Lett. B*154 (1985) 407;
J. Kunz, and Y. Brihaye, Fermions in the background of the sphaleron barrier, *Phys. Lett. B*304 (1993) 141.
- [26] M. Axenides, A. Johansen, H. B. Nielsen, O. Törnkvist, \mathbf{Z}_2 as the topological origin of B+L violation in the hot electroweak theory, *Nucl. Phys. B*474 (1996) 3.

n	k	E	$\varepsilon(x=0)$	ε_{max}	$\varphi(x=0)$
1	1	0.804	12.48	12.48	-1.711
1	2	0.966	636.2	636.2	-3.374
1	3	0.994	25352.	25352.	-5.154
1	4	0.999	968441.	968441.	-6.968
2	1	1.336	0.5457	1.734	-1.399
2	2	1.773	7.256	23.08	-2.430
2	3	1.927	77.25	251.0	-3.524
2	4	1.977	840.2	2674.	-4.673
3	1	1.800	0.192	0.92256	-1.279
3	2	2.482	1.629	7.552	-2.110
3	3	2.785	10.79	49.94	-2.951
3	4	2.913	69.88	322.4	-3.837
4	1	2.231	0.0941	0.6541	-1.209
4	2	3.137	0.6388	4.132	-1.943
4	3	3.588	3.249	20.52	-2.649
4	4	3.808	15.92	99.91	-3.387

Table 1

The energy E (in units of $\frac{4\pi}{\kappa g}$), the energy density at the origin $\varepsilon(x=0)$, the maximum of the energy density ε_{\max} and the dilaton field at the origin $\varphi(x=0)$ are shown for the solutions with node numbers $k = 1 - 4$ for the sequences $n = 1 - 4$.

$$\epsilon(\rho, z) \quad n=3 \quad k=3$$

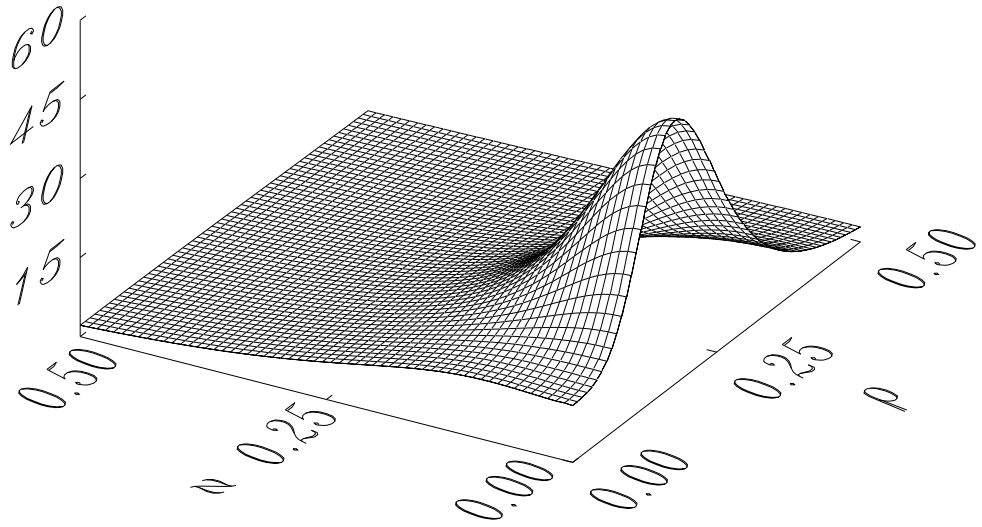


Figure 1: The energy density $\epsilon(\rho, z)$ (in units of $\frac{4\pi}{\kappa g}$) is shown as a function of the dimensionless coordinates ρ and z for the multisphaleron with winding number $n = 3$ and node number $k = 3$

$$F_1(\rho_c, z_c) \quad n=3 \quad k=3$$

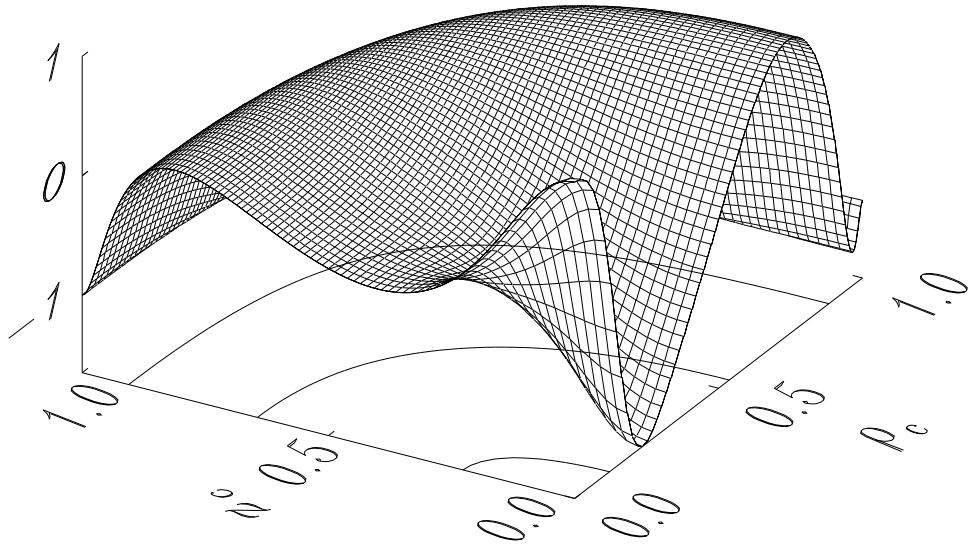


Figure 2a: The gauge field function $F_1(\rho_c, z_c)$ is shown as a function of the dimensionless coordinates ρ_c and z_c for the multisphaleron with winding number $n = 3$ and node number $k = 3$. The coordinates ρ_c and z_c are defined by $\rho_c = \frac{x}{1+x} \sin \theta$ and $z_c = \frac{x}{1+x} \cos \theta$, respectively. The contourlines indicate the nodes of the function.

$$F_2(\rho_c, z_c) \quad n=3 \quad k=3$$

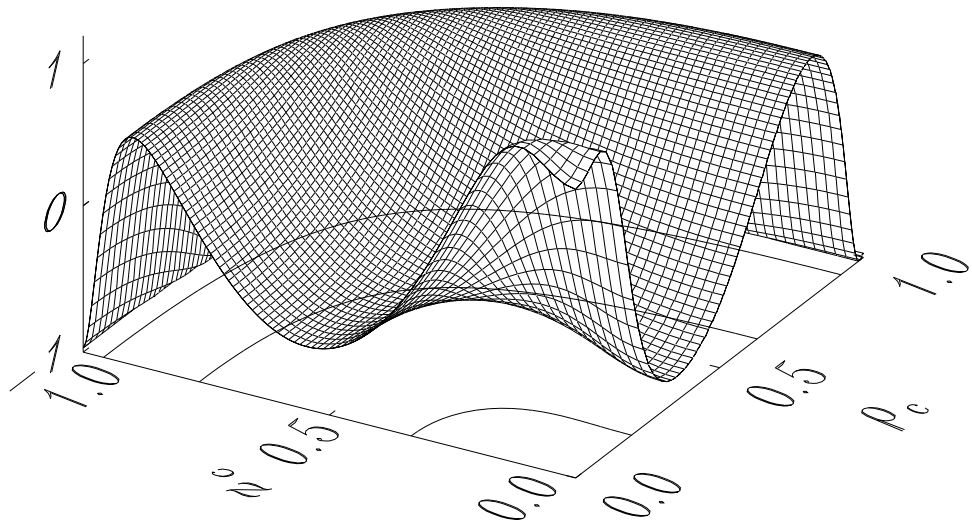


Figure 2b: The same as Figure 2a for the gauge field function $F_2(\rho_c, z_c)$.

$$F_3(\rho_c, z_c) \quad n=3 \quad k=3$$

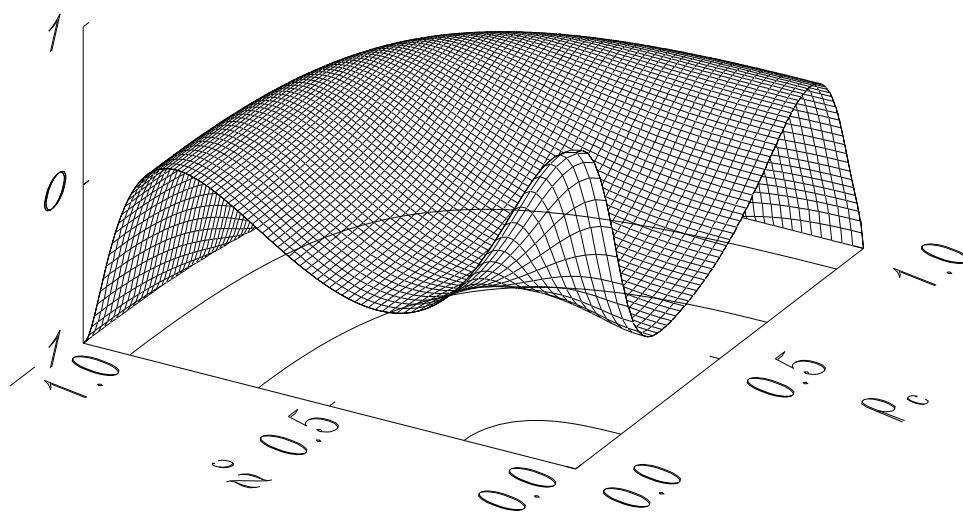


Figure 2c: The same as Figure 2a for the gauge field function $F_3(\rho_c, z_c)$.

$$F_4(\rho_c, z_c) \quad n=3 \quad k=3$$

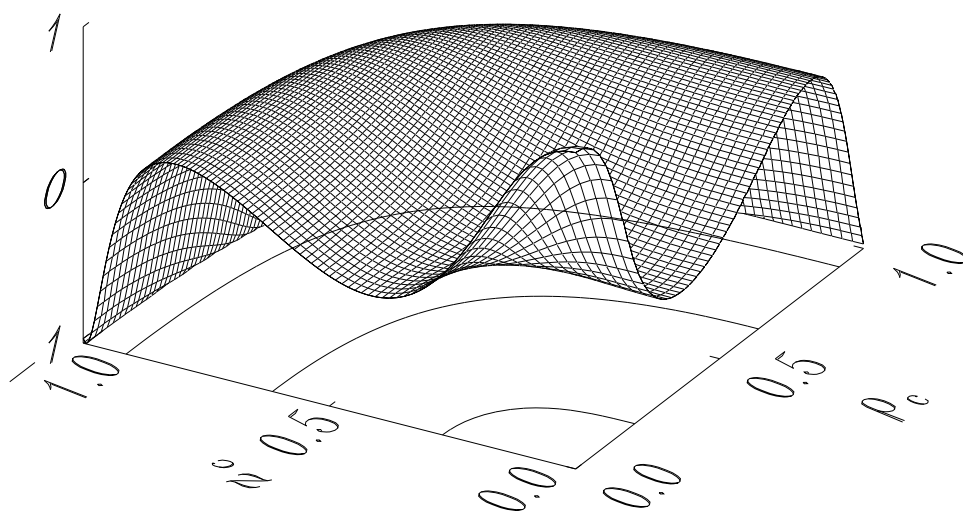


Figure 2d: The same as Figure 2a for the gauge field function $F_4(\rho_c, z_c)$.

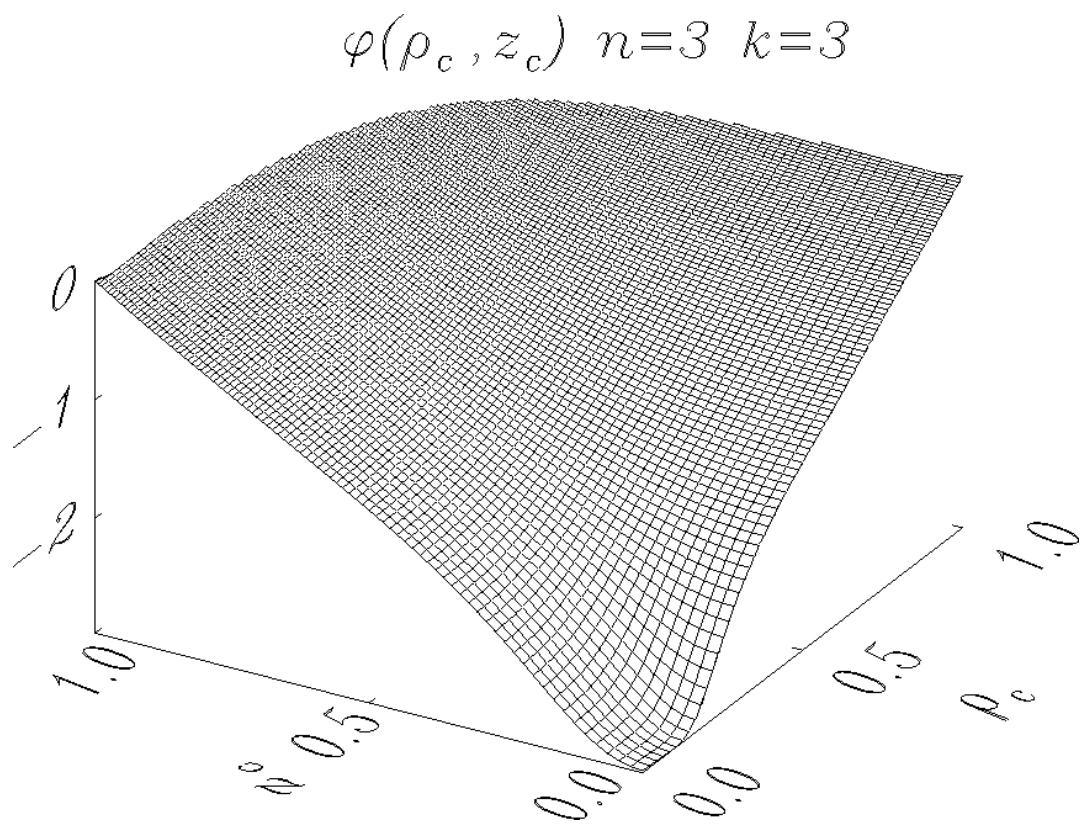


Figure 3: The same as Figure 2a for the dilaton function $\varphi(\rho_c, z_c)$.

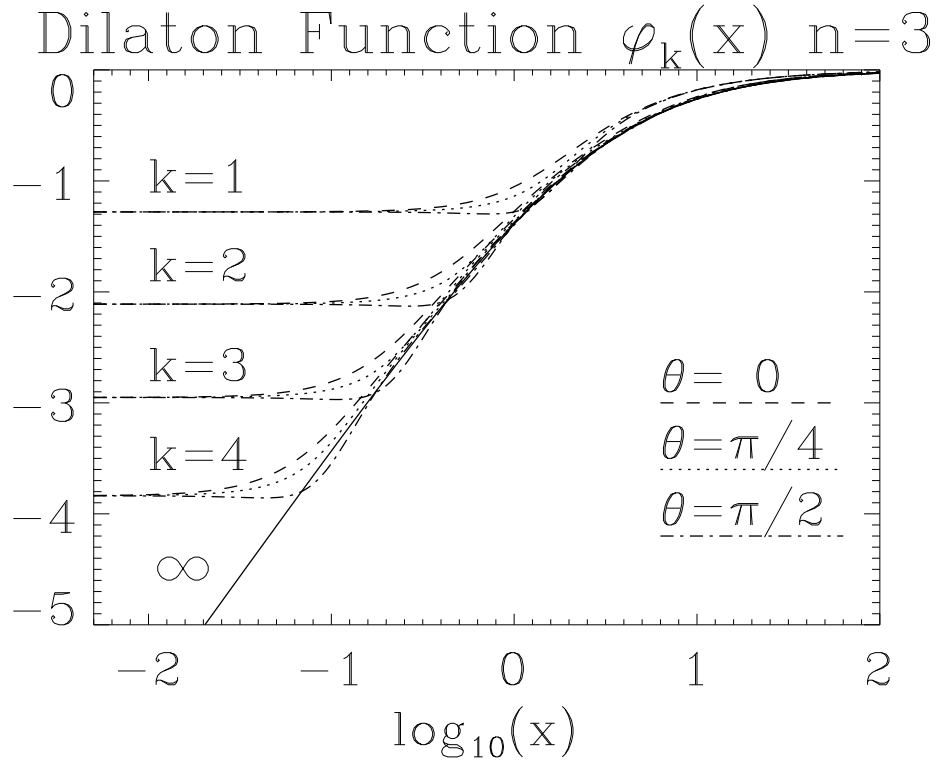


Figure 4: The dilaton functions $\varphi_k(x, \theta)$ for the multisphalerons with winding number $n = 3$ and node numbers $k = 1 - 4$ are shown as a function of the dimensionless coordinate x . The dashed, the dotted and the dash-dotted lines represent the angles $\theta = 0$, $\theta = \pi/4$ and $\theta = \pi/2$, respectively. Also shown is the limiting function $\varphi_\infty(x)$ (solid line).